

ANOMALOUS ISOTOPE SHIFTS IN Pb NUCLEI IN RMF THEORY

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Abstract

We have studied the anomalous behaviour of isotopic shifts of Pb nuclei in the relativistic mean field theory. It has been shown that the relativistic mean field provides an excellent description of the anomalous kink in the isotopic shifts about ^{208}Pb . This is in contrast from density-dependent Skyrme forces which do not reproduce the observed trend in the empirical data on the charge radii. We discuss some differences in the description of isotope shifts in the RMF theory and the Skyrme mean field.

1. Introduction

The relativistic mean field (RMF) theory^{1,2)} has of late been very successful in describing the ground-state properties of nuclei at and away from the stability line. This is in contrast to phenomenological density dependent Skyrme forces³⁾ where the description of nuclei is constrained mainly to the stability line. Moreover, the spin-orbit interaction arising from the Dirac description of nucleons provides an attractive feature of the RMF theory. In the non-relativistic Skyrme theory on the other hand, the spin-orbit interaction is added only phenomenologically. The saturation mechanisms of the two theories are evidently quite different. In the RMF theory the saturation and the density dependence of the nuclear interaction stems from a balance between large attractive scalar σ -meson field and large repulsive vector ω -meson field. The asymmetry component is accounted for by the isovector ρ meson. In essence, the nuclear interaction is generated by the exchange of various mesons between nucleons. This leads to the

structure of the force in the RMF theory and consequently a density dependence which differs from that of Skyrme theory. In the Skyrme ansatz, however, the density dependent term is phenomenological and is obtained by fitting the properties of nuclei.

The charge radii of Pb isotopes and their isotope shifts have been investigated in detail⁴⁾ using density-dependent Skyrme forces. The isotopic chain of Pb nuclei is known to exhibit a kink in the empirical isotope shifts⁵⁾ about shell-closure. This implies that heavier isotopes obtained on adding further neutrons to ^{208}Pb show unusually large charge radii as compared to the lighter ones. The Skyrme forces do succeed in describing the isotope shifts and thus charge radii of nuclei only on the lighter side of ^{208}Pb , where a density-dependent pairing force is required to be switched on. The isotope shifts of the heavier nuclei, can not, however, be described by any of the standard Skyrme forces, as has been discussed in detail in ref.⁴⁾. Even including possible ground-state correlations does not improve the description. Thus, the Skyrme mean field is not able to describe the charge radii of heavier isotopes of Pb. In this work, we investigate the long-standing problem of the isotope shifts in the RMF theory. In sect. 2 we present briefly some features of the RMF theory. In sect. 3 our results are presented and discussed.

2. The RMF theory approach

The ansatz of the interaction in the RMF theory is based upon Lagrangian density of the form¹⁾:

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(i\not{\partial} - M)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma) - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} \\ & + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\vec{\rho}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - g_\sigma\bar{\psi}\sigma\psi - g_\omega\bar{\psi}\not{\omega}\psi - g_\rho\bar{\psi}\not{\vec{\rho}}\vec{\tau}\psi - e\bar{\psi}\not{A}\psi. \end{aligned} \quad (1)$$

where the Dirac nucleon interacts with the σ and ω meson fields. The ρ meson generates the isovector component of the force. The nonlinear $\sigma\omega\rho$ model which we use, has a nonlinear scalar self-interaction of the σ mesons as given by

$$U(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4, \quad (2)$$

where g_2 and g_3 are the non-linear parameters. Details on the RMF theory have been discussed in ref.²⁾. The parameter sets NL1⁶⁾ and NL2⁷⁾ have been used extensively to obtain properties of nuclei. It was shown by Sharma and Ring⁸⁾ that both the above forces provide neutron skin thickness of neutron-rich nuclei much larger than the empirical values. Investigating the

ground-state properties of nuclei⁹⁾ in the non-linear $\sigma\omega\rho$ model, it was noted that indeed a stronger ρ meson coupling and therefore a very large asymmetry energy of the above forces has been responsible for larger neutron skin thickness of neutron-rich nuclei. Consequently, a new force NL-SH was obtained, where the above problem of the earlier forces was resolved. It was also shown⁹⁾ that this force describes very well the ground-state binding energies, charge and neutron radii of spherical nuclei near the stability line as well as those of deformed nuclei very far off the stability line. Here, we study isotope shifts in the RMF theory using the forces NL1 and NL-SH.

3. Results and discussion

We have performed calculations within the Hartree approximation. Although most of the Pb isotopes close to ^{208}Pb are spherical, an axially symmetric configuration has been assumed and Hartree minimization has been performed. The method of the oscillator expansion¹⁰⁾ has been employed, whereby both the fermionic as well as bosonic wavefunctions have been expanded in $N = 12$ shells. We have considered all the even-mass Pb isotopes from $A = 190$ to 214 . For convergence reasons, $N = 14$ Fermionic shells have also been considered. It is found that the difference between the $N=12$ and $N=14$ calculations is very small. Therefore, only the results obtained with $N=12$ are presented. For all the open-shell nuclei pairing has been included within the BCS formalism. The pairing gaps have been obtained from the particle separation energies of the neighbouring nuclei. The quadrupole deformations obtained from the convergence are very small. These are practically close to spherical configuration for all the nuclei we have considered.

The binding energy per nucleon of Pb isotopes obtained in the RMF theory are shown in Fig 1a. The empirical binding energies are also shown for comparison. The binding energies of all isotopes are reproduced well by the set NL-SH. The deviations are at most 0.1%. Here we also compare the results obtained with NL1. For the lighter nuclei of the isotopic chain, the binding energies obtained with NL1 show a systematic deviation from the empirical data. The difference between the calculated and the empirical binding energies shows an increase as the neutron excess decreases. This discrepancy is due to the asymmetry energy of about 44 MeV for NL1, which is larger than the empirical value.

The calculated charge radii have been used to obtain the isotope shifts. The nucleus ^{208}Pb has been taken as the reference point. In order to provide a good illustration, the isotope shifts ($\Delta r_c^2 = r_c^2(A) - r_c^2(208)$) have been modified by subtracting an equivalent of the liquid-drop

difference ($\Delta r_{LD}^2 = r_{LD}^2(A) - r_{LD}^2(208)$) obtained from $r_{LD}^2(A) = \frac{3}{5}r_0^2 A^{2/3}$ for Pb nuclei, as in ref.⁴⁾. All the results are presented in the same way. The empirical values are from the precision data obtained from the atomic beam laser spectroscopy⁵⁾. The empirical data exhibit a conspicuous kink about ^{208}Pb . The figure also shows the theoretically obtained isotope shifts for the two forces NL-SH and NL1. We compare them with those from Skyrme interaction SkM*. The isotope shifts from NL-SH reproduce the kink very well¹¹⁾. It is only below $A = 198$ that the theoretical isotope shifts show a divergence from the empirical data. This behaviour is much below the kink and might be attributed to the transitional behaviour of nuclei in the light Pb isotopes, which is accounted for in our theory. This region of mass number usually encounters such effects. The force NL1, on the other hand, also shows a reasonable kink on the higher side of ^{208}Pb . On the lower side, however, NL1 shows a slight divergence from the data and the slope of the theoretical values is also different from that of the empirical data. It may be noted that due to inaccurate description of the other ground-state properties and a very large asymmetry energy, NL1 is not expected to describe the isotope shifts adequately. In the same figure we also show for comparison isotope shifts for SkM* as taken from ref.⁴⁾. Only the data points on ^{194}Pb and ^{214}Pb are shown, which are representative of the behaviour of SKM*. On the lighter side, SkM* shows a behaviour similar to NL-SH. SkM*, however, shows an almost linear function with mass number and consequently displays a clear divergence from the empirical data on heavier side.

The kink in the experimental data implies that adding neutrons to the closed neutron-core of 126 neutrons changes the mean field of protons which brings about this kink. Attempts have been made to reproduce this kink using the density-dependent Skyrme forces. This has been discussed in detail in ref.⁴⁾. As shown in fig. 2 of ref.⁴⁾, all the Skyrme forces e.g. SkM*, Ska and SIII show a strong deviation from the empirical data in the isotope shifts for nuclei heavier than ^{208}Pb . The lighter Pb isotopes could, however, be described by SkM* and SGII. The binding energies, on the other hand, show a behaviour opposite to that of isotope shifts. For example, SkM* reproduces the binding energies of isotopes including and heavier than ^{208}Pb . However, it shows a systematic divergence from empirical binding energies for lighter isotopes. The disagreement increases on going to the neutron-deficient side. The other two Skyrme forces show disagreements with the empirical binding energies on both the sides of the closed shell. Including all possible corrections beyond the mean field does not cure the problem. As shown in ref.⁴⁾, Skyrme forces are unable to describe the binding energies as well as isotopes shifts of nuclei away from ^{208}Pb .

The ability to reproduce the kink by the RMF theory and the failure of the present Skyrme interactions not to be able to do so, raises some important questions. The shell effects that are inherent in the structure of nuclei over the periodic table are described differently by various theories. The kink in the empirical data on the charge radii of Pb isotopes is one of the aspects which has remained hitherto unsolved. Presently, the RMF theory succeeds in accommodating these shell effects which run across the shell-closure. The shell effects across the magic numbers play a significant role in astrophysical r-processes. It should also be noted that a recent study of nuclei away from stability line has shown significant differences in the shell effects predicted by the two approaches¹²⁾.

The behaviour of the Skyrme mean-field approach and of the RMF theory towards isotope shifts indicates an important difference between the two approaches. This difference is not easy to explain. However, examining the two approaches we feel that the basic density dependence of the two interactions and the difference in the saturation mechanism of the two methods could be at the origin of the difference in the isotope shifts. Our calculations have shown that the kink originates from the collective contribution of many single-particle orbitals to the proton rms radius. The main contribution comes from the outer orbitals. Another important aspect that is different is the spin-orbit interaction as mentioned earlier. Whereas in the RMF theory, the spin-orbit interaction originates from the coupling of σ and ω mesons to Dirac nucleon, the spin-orbit term in the Skyrme approach is added phenomenologically. Our study has also shown a slightly different sequence of the single-particle levels which should be attributed to the spin-orbit splitting differences in the two approaches. The spin-orbit splitting is responsible for putting different orbitals in space and thus determining the structure of a nucleus. Thus, a difference in the spin-orbit splitting in the two methods would contribute to the difference in the isotope shifts of the two approaches.

Very recently it has also been shown that a non-relativistic reduction of the relativistic Hamiltonian leads to a spin-orbit potential which exhibits a different isospin-dependence compared with the Skyrme spin-orbit potential¹³⁾. It has turned out, that an isospin-dependent spin-orbit term in the Skyrme mean field approach can produce reasonable values for the kink, too. As an example we show in fig. 2 the experimental values of the isotope shifts in the vicinity of the shell closure and compare them with SkM* results for two sets of isospin-admixtures in the spin-orbit term:

$$W_{\tau}(r) = W_1 \nabla \rho_{\tau} + W_2 \nabla \rho_{\tau' \neq \tau} . \quad (3)$$

$W_1 = 2 W_2$ corresponds to the standard (isospin independent) spin-orbit and $W_1 = 1.05 W_2$ has a strong isospin dependence. All the other parameters, in particular W_2 , are the same as in SkM* force in both cases. Further work in this direction is in progress.

This work is supported by the E.U. HCM programme, contract: EG/ERB CHBICT-930651

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Figure Caption

- Fig. 1. On the left side (Fig. 1a) the binding energies of Pb isotopes obtained with the forces NL1 and NL-SH together with the empirical values (expt.). On the right (Fig. 1b) the isotope shifts with the same forces. The SkM* values⁴⁾ and the empirical data are also shown.
- Fig. 2. The isotope shifts of Pb nuclei for two sets of isospin admixture in the spin-orbit term of the Skyrme force SkM*.

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